Let $z = x^2+y^2$

Find the rate of change of z at $(1,1)$

a) in x-direction

$$\frac{\partial z}{\partial x} = 2x\Big|_{(1,1)} = 2$$

b) in y-direction $\frac{\partial z}{\partial y} = 2y\Big|_{(1,1)} = 2$

c) in $\vec{V}$, where $\vec{V} = \langle 1, 2\rangle \rightarrow \bar{u}_v = \langle \frac{1}{\sqrt5}, \frac{2}{\sqrt5}\rangle$

$$\lim_{h\to 0} \frac{f(x+\frac{1}{\sqrt5}h, y+\frac{2}{\sqrt5}h) - f(x,y)}{h}$$

$$= \lim_{h\to 0} \frac{\left(x+\frac{h}{\sqrt5}\right)^2 + \left(y+\frac{2}{\sqrt5}h\right)^2 - (x^2+y^2)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2h}{\sqrt5}x + \frac{h^2}{5} + \frac{4h}{\sqrt5}y + \frac{4h^2}{5}}{h} = \lim_{h\to 0} \frac{2x}{\sqrt5} + \frac{h}{5} + \frac{4y}{\sqrt5} + \frac{4h}{5}$$

$$= 2x\cdot\frac{1}{\sqrt5} + 2y\cdot\frac{2}{\sqrt5} = \langle 2x, 2y\rangle \cdot \langle \frac{1}{\sqrt5}, \frac{2}{\sqrt5}\rangle$$

$$= \underbrace{\langle f_x, f_y\rangle}_{\text{gradient vector}} \cdot u_v$$

$$D_u\left(f(x,y)\right) = \nabla f \cdot u \quad , u = \langle a, b\rangle$$

$$= \langle f_x, f_y\rangle \cdot \langle a, b\rangle$$

$$z = f(x,y) \qquad D_u(z) =$$

$$\lim_{h\to 0} \frac{f(x+ah, y+bh) - f(x,y)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+ah, y+bh) - f(x, y+bh) + f(x, y+bh) - f(x,y)}{h}$$

$$= \lim_{h\to 0}\left(\frac{f(x+ah, \underline{y+bh}) - f'(x, \underline{y+bh})}{ah}\right)a + \underbrace{\lim_{h\to 0}\left(\frac{f(x, y+bh) - f(x,y)}{bh}\right)b}$$

$$= \quad f_x \cdot a \qquad + f_y \cdot b$$

b